Me 421 - Fall 2021

Shaft Design and Fabrication

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Professor Simmons

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We have adhered to the Duke Community Standard in completing this assignment. We understand that a violation of the Standard can result in failure of this assignment, failure of this course, and/or suspension from Duke University.

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1 Executive Summary

The purpose of this memo is to provide an update on the prototype tensioner shaft design for a packaging machine drive. The team, tasked with developing the final product for manufacturing, was working within several constraints. The shaft must be operated at between 100-300 RPM and will be in continuous use except during maintenance. Additionally, the shaft must apply a constant of 100 lbf to tension the drive belt, which is attached via a timing belt sheave. Lastly, the shaft must be machined from a ferrous alloy, although our prototype was made from aluminum for easy machining.

The final shaft, constructed from AISI 1045 steel, is designed for infinite life and has a designed factor of safety of 4.8. Excluding the ball bearings, each unit costs less than $ 90 to manufacture. The shaft has a final length of 7.625" and is machined from stock with a 1" diameter. Additionally, our shaft accommodates a Timken S8k ball bearing as well as a pillow-block bearing on either side. A quick-disconnect (QD) bushing was implemented to fix the timing belt sheave to our shaft, allowing for easy maintenance and interchange of parts in case of malfunction. Our design decisions, fatigue calculations, and cost analysis are described in further detail throughout this memo.

2 Shaft Design

2.1 Design Calculations

Analyzing a free body diagram of the shaft with a tension of 100lbf on the belt, the shaft was assumed to have a bending force of 200lbf on its center. Thus, the bending stress was analyzed by creating Shear and Moment diagrams for the shaft.

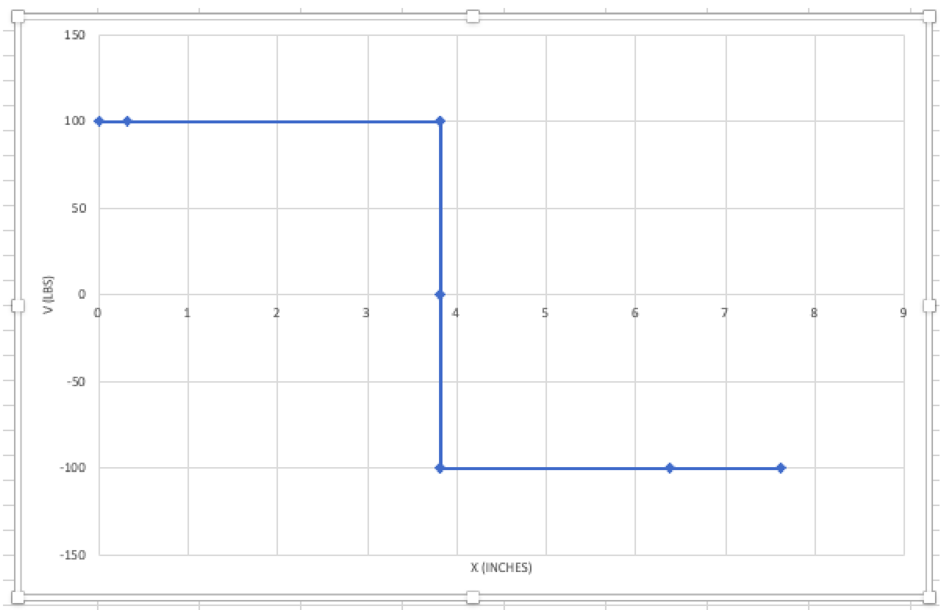

Figure 1: Shear Diagram

The bending stress equation was used to analyze bending stress at each of the shoulders and the center of the shaft.

$$\sigma = \frac{Mc}{I} \tag{1}$$

At the first shoulder, M = 31 lbf, c = 0.375", I = $\frac{\pi(0.375)^4}{4}$ = 0.0155

$$\sigma = \frac{Mc}{I} \tag{2}$$

$$\sigma_{shoulder1} = 748.48 \tag{3}$$

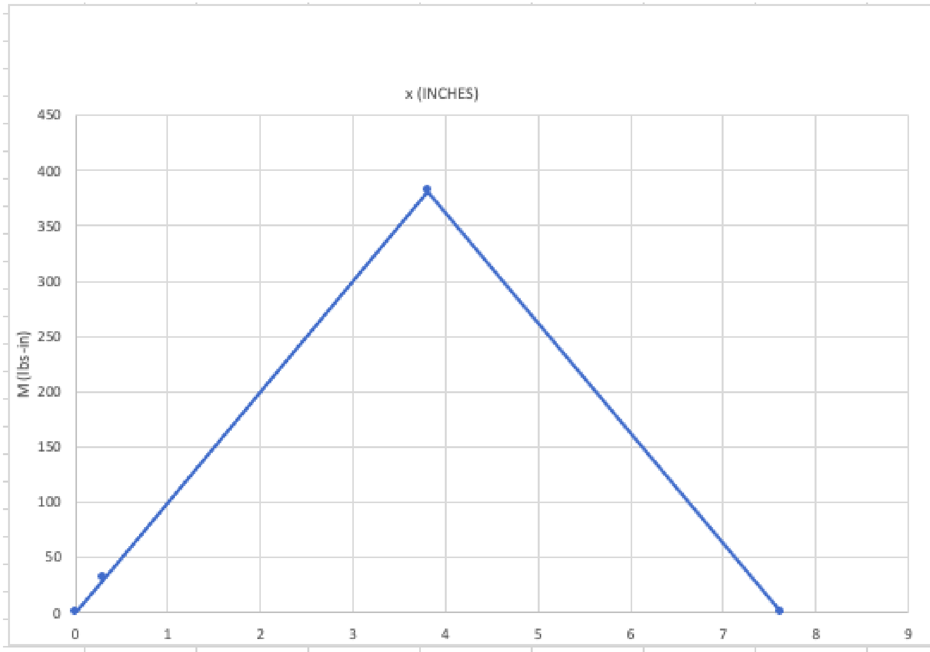

Figure 2: Moment Diagram

At the center of the shaft: M = 381.25 lbf, c = 0.495", I = $\frac{\pi(0.495)^4}{4}$ = 0.0472

$$\sigma = \frac{Mc}{I} \tag{4}$$

$$\sigma_{center} = 4002.26psi \tag{5}$$

At the second shoulder, M = 127.08 lbf, c = 0.375", I = $\frac{\pi(0.375)^4}{4}$ = 0.0155

$$\sigma = \frac{Mc}{I} \tag{6}$$

$$\sigma_{shoulder2} = 3074.52psi \tag{7}$$

Thus, the maximum bending stress on the shaft occurs at the center and the maximum bending stress is 4002.26psi. For AISI 1045, the yield strength is 77000psi. Our factor of safety is for bending of the shaft is:

$$F.S. = \frac{\sigma}{\sigma_Y} \tag{8}$$

$$F.S. = \frac{77000}{4002.26} = 19.24 \tag{9}$$

2.1.1 Tolerance

LT1 (Transitional Location Fit 1): +/- .00025”

2.1.2 Factor of Safety

F.S. (final with fatigue) = 5.198

2.2 Component Selection

Initially, the group designed a shaft with far more components than necessary. Specifically, a shoulder and snap ring was used to hold the bushing in place in the center of the shaft and extra machining was proposed to be performed on the shaft to ease assembly. Ultimately, after learning more about the functionality of the bushing and the way it tightens as the sheave is placed on the outside of the bushing, the team realized that the extra step in the shaft as well as the axial retaining methods were unnecessary to ease assembly or hold the part in place and only added to cost. The interference pressure of the bushing with the sheave attached is more than enough to hold the bushing in axially and without the sheave attached, the bushing slides easily along the shaft. This initial design is shown in the appendix in figure 6. When designing the tensioner shaft for a packaging machine drive, several of the components had been pre-determined. At a length of 7.625" and diameter of 1”, the shaft has two bearings on either end. The first is a Timken ball bearing P/N S8K – an inner diameter of 0.75” and a width of 0.3125”. Therefore, on one end of the shaft there is a shoulder that allows for this bearing to be affixed. The purpose of the ball bearing is to facilitate rotary movement between the shaft and another machinery component, such as a bracket or hub. This is achieved with two ‘races’ that are separated by balls, allowing for a reduction of friction and surface contact. For this to be of use, therefore, the inner race must be appropriately secured to the shaft. In other words, the bearing must be able to slide onto the shaft, but once on the fit should be sufficient to not allow any rotary slip. Hence, to satisfy these two requirements an interference fit was used, leading to a shaft diameter of 0.7505”.

The specifications also require for a pillow-block bearing to be attached on the other end. This too has an inner diameter of 0.75” but a wider width of 1.25” leading to a longer shoulder having to be cut. The pillow-block bearing is used to provide support to the rotating shaft and anchor it to a stationary surface; in other words, the outer ring is stationary, and the inner ring is rotating. Similar to the ball bearing, this setup requires a tight fit between the shaft and the bearing – an interference fit means that the shaft here is therefore machined to a diameter of 0.7505”.

While there is often concern for radial motion when utilizing bearings, in this case, the group assumed that the Timken S8K bearing would also be secured somehow in another mount, thus fixing both bearings in place. With both bearings fixed in place, the shoulders on the inside of each respective bearings prevent motion toward the outside of the bearings. For example, the shoulder on the right side of the shaft prevents the shaft from moving to the right and vice versa for the left bearing. Ultimately, the combination of the shoulders on either side of the shaft produce a fully located shaft in either radial direction and thus there is no need for any other component to secure the bearings radially, such as a snap ring or collar.

The final part of the shaft to design is adding the timing belt sheave. This particular component is attached by using a bushing, which rigidly connects to the sheave and is fitted onto the shaft. The SDS style bushing is automatically chosen as a result of the 4” 32-teeth timing belt sheave specified in the requirements. The inner diameter could be chosen from a couple of options. Due to the fact that the shaft comes in a 1" diameter, to reduce machining and therefore cost, the 1" bushing was selected. The position of the bushing on the shaft has also been pre-determined, so the remaining aspects to be researched are whether additional features are needed to keep the bushing in place. Firstly, to limit axial movement, the system was analysed to see if snap rings or a collar could be required – these are effective features, particularly when strong axial forces are expected and the component in question is not firmly fastened. In this case, however, both of those instances are false. Indeed, the bushing is designed with a slightly wider diameter than the sheave; therefore, when collated on the shaft, the sheave causes the bushing’s inner diameter to contract around the shaft causing an interference fit. Compounding this consideration with the fact that very little axial forces are expected, no additional feature was added to the design since the fit’s pressure will sufficiently restrict movement. In contrast, far more radial forces will be subjected from the drive belt. Consequently, as explained further in the Fatigue Failure section, a key was added to the design to reinforce the bushings position on the shaft and to reduce any slip.

2.3 Material Selection

Although several steel alloys were considered for this application, the team chose high-strength AISI 1045 steel for our final design. An AISI 1045 shaft has a yield strength of 77000 psi and is widely used in engineering projects, including shafts, nuts, and studs. This steel, when compared to other steels, is of high strength, easily machinable, and cheap. AISI 1045 stock is also widely available, with a 12” shaft of 1” diameter costing $10 on McMaster-Carr. The balance of strength, cost, and machinability made high strength AISI 1045 steel the optimal material choice for our tensioner shaft. A drawback of this steel is its lack of corrosion resistance that can be found in other steels, either those with high chromium levels or those with chrome plating. However, the intended environment for the shaft does not require heightened levels of corrosion resistance and therefore this resistance is unnecessary. Additionally, these steels are much harder to machine and cost more than AISI 1045 steel, making them unfeasible for this application.

2.4 Cost Analysis

2.4.1 Bill of Materials

Table 1: Bill of Materials

Part	Description	Qty	Unit Cost ($)	McMaster-Carr Part #
Shaft	AISI 1045 Steel Shaft	1	10.04	8924K1
Timken Ball Bearing	Used to reduce friction and locate the shaft	1	8.39	60355K507
Pillow Block Bearing	Used to mount and reduce friction while the shaft rotates	1	86.85	7728T53
Timing Belt Sheave	The pulley that attaches to the timing belt to transmit torque to/from the shaft	1	54.51	6495K218
Bushing (Including Clamping Screws and Set Screws)	SDS style Quick-Disconnect Bushing to attach timing belt sheave to the shaft	1	24.4	6086K316
			($): 184.19	

3 Fatigue Failure and Estimated Life

AISI 1045 steel has a Young's Modulus (E) of 77 Mpsi, Yield Strength (S_y) of kpsi, and a Tensile Strength (S_{ut}) of 91 kpsi[1].

To calculate the endurance limit of the shaft, the Marin equation below was used.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{10}$$

The rotating beam specimen endurance limit (S'_e) was estimated as half of the tensile strength or 38.5 kpsi. With the help of Table 6.2 in Shigley's, the surface condition modification factor (k_a) was calculated to be 0.817. The size factor (k_b) was determined to be 0.8799, while the loading factor (k_c) and the temperature factor (k_d) were set to 1. The reliability modification factor (k_e) was also taken into consideration since most endurance strength data are reported as mean values. Thus, for 99 % reliability, k_e is 0.814. Lastly, the Miscellaneous Effects factor (k_f) was not considered. Using the the Marin equation, the endurance limit of the shaft was estimated as,

$$S_e = 22.53 kpsi. \tag{11}$$

In addition to the endurance limit, stress concentration factors were considered. Using a fillet radius of 3mm or 0.118 in, an outer and inner shoulder diameter of 0.75 in and 0.99 in respectively, the notch sensitivity (q) of about 0.9 was determined using Figure 6-20 in Shigley's. The stress concentration factor K_t of 1.45 was then found using figure A-15-9 in Shigley's. Using these parameters, the fatigue stress- concentration factor was determined from equation

$$K_f = 1 + q(K_t - 1) \tag{12}$$

$$K_f = 1.41 \tag{13}$$

To estimate the maximum cycles to failure, the maximum reversing stress needed to be considered:

$$(\sigma_{rev})_{max} = K_f(\sigma_{rev})_{nom} = 128.31kpsi. \tag{14}$$

Taking f to be 0.85,the constants a and b are found with the following formulas:

$$a = \frac{fS_{ut}^2}{S_e} = 265.56kpsi \tag{15}$$

$$b = \frac{-1}{3}log\frac{fS_{ut}}{S_e} = -0.178 \tag{16}$$

Finally, the maximum cycles till failure was found to be

$$N = (\frac{\sigma_{rev}}{a})^{\frac{1}{b}} = 6.99 * 10^{10}cycles \tag{17}$$

The fatigue factor of safety (n) was then found to be

$$n = \frac{S_e}{K_f \sigma_{rev}} = 5.198 \tag{18}$$

In another convention, steel shafts are estimated to have an infinite life as long as they are loaded under half of their yield strength. In this case, 1045 carbon steel has a yield strength of about 45,000 psi, which is well above double the loaded stress of the shaft. In theory, this means the designed shaft will have an infinite life and that the components, like the bearings and the key, will fail long before the shaft itself. This assumption lines up with the fatigue analysis performed on the shaft which stated the the lifetime of the shaft would be around $6.99 * 10^{10}$ cycles.

4 Additional Concerns

Another failure mode of the shaft can stem from bearing failure. This bearing failure will likely occur as a result of spallation after a large number of cycles, granted the bearing can be kept free of debris. Given these conditions, Timken, the bearing manufacturer, has calculated a 90% probability that the bearing will last through 1 million cycles with a load of 2340 lb_f. The bearings in use in this case are only under 200 lb_f, thus they are expected to last longer. While calculations can be performed to find this bearing life, Timken includes a bearing life calculator on their website in order to find this information. The inputs to this calculator include speed, which the team decided to rate at the top speed (300 rpm) in order to discover the minimum bearing life, axial load (200 lb_f) and operating temperature which the group assumed to be room temperature. Ultimately, the life of the two bearings in this case were calculated to have a 90% probability to still be in functioning form after 41300 hours. This lifetime far exceeds what is necessary for a machine of this sorts to need maintenance and thus these bearings fit this use very well [3].

5 Appendix

5.1 A.1: Final Shaft Design

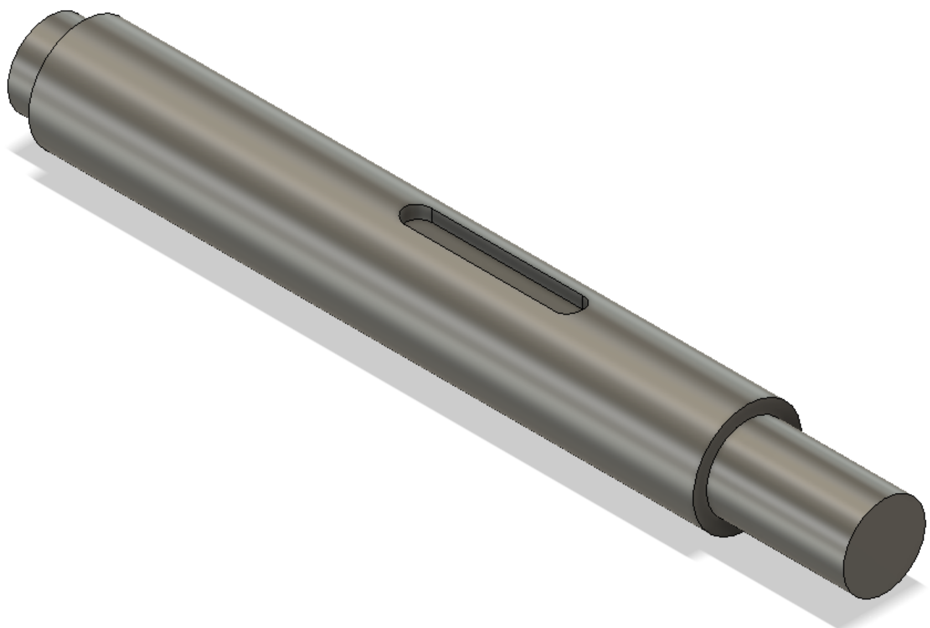

Figure 3: Final Shaft Design

5.2 A.2: Final Shaft Assembly

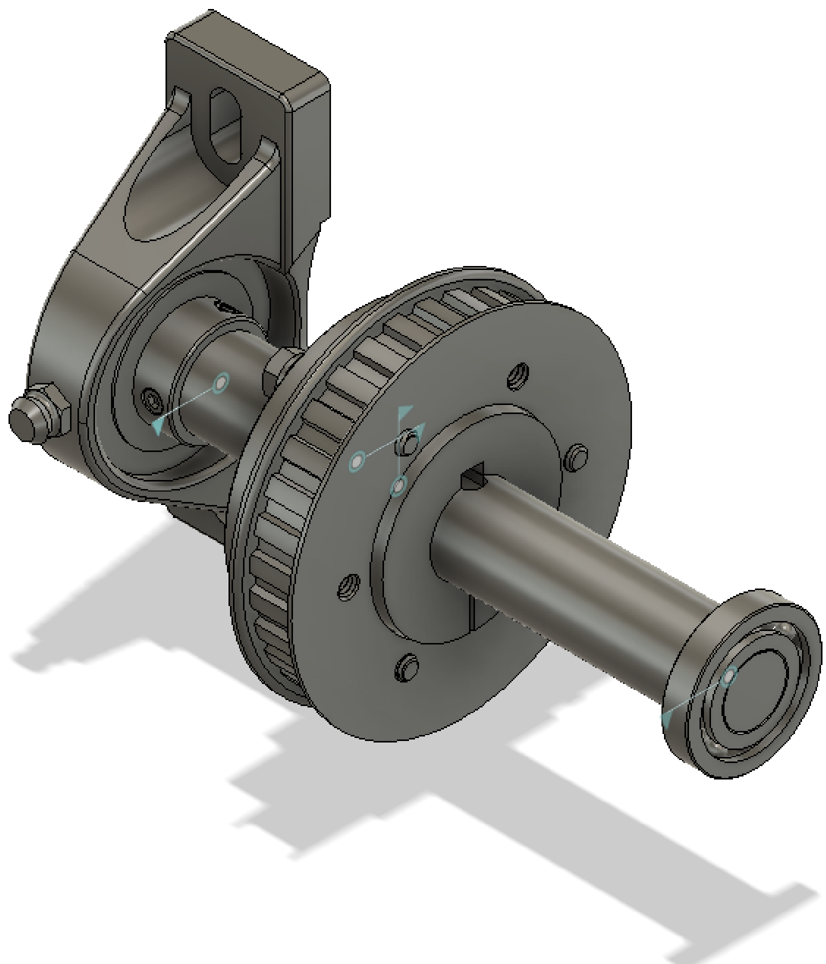

Figure 4: Final Shaft Assembly

5.3 A.3: Final Shaft Drawing

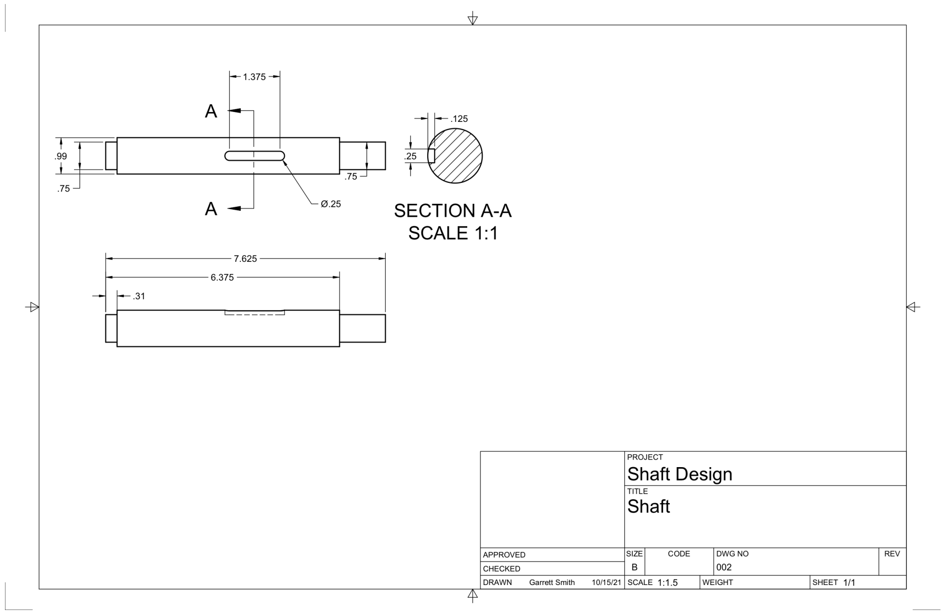

Figure 5: Final Shaft Drawing

5.4 A.4: Initial Shaft Design

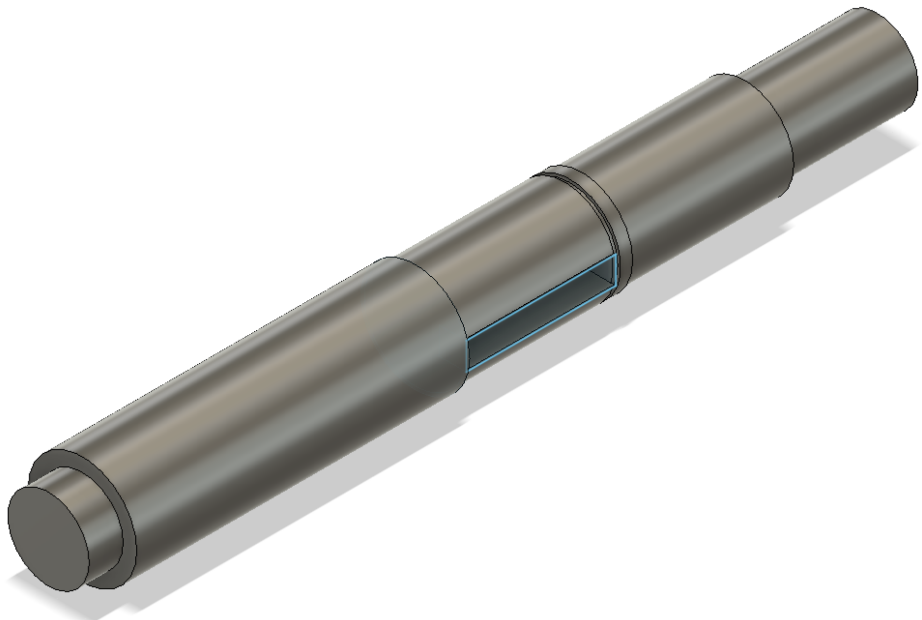

Figure 6: Initial Shaft Design

5.5 A.5: Initial Shaft Drawing

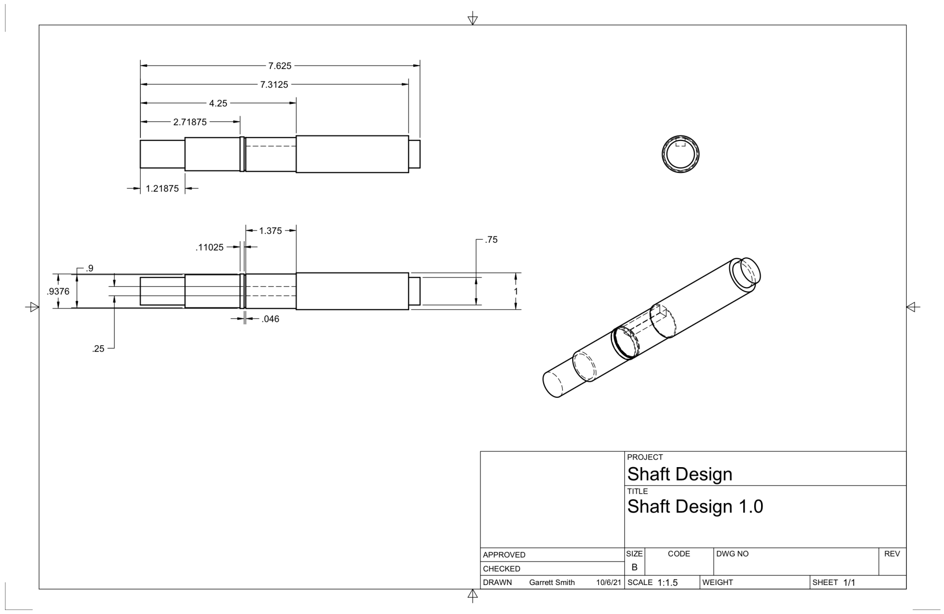

Figure 7: Initial Shaft Drawing

5.6 A.6: Manufactured Shaft Prototype

Figure 8: Manufactured Shaft Prototype

6 References

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